

Linear square-mass trajectories of radially and orbitally excited hadrons in holographic QCD

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ABSTRACT: We consider a new approach towards constructing approximate holographic duals of QCD from experimental hadron properties. This framework allows us to derive a gravity dual which reproduces the empirically found linear square-mass trajectories of universal slope for radially and orbitally excited hadrons. Conformal symmetry breaking in the bulk is exclusively due to infrared deformations of the anti-de Sitter metric and governed by one free mass scale proportional to Λ_{QCD} . The resulting background geometry exhibits dual signatures of confinement and provides the first examples of holographically generated linear trajectories in the baryon sector. The predictions for the light hadron spectrum include new relations between trajectory slopes and ground state masses and are in good overall agreement with experiment.

KEYWORDS: QCD, Nonperturbative Effects, Gauge-gravity correspondence, Confinement.

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1. Introduction

Since the inception of Quantum Chromodynamics (QCD), progress in understanding its low energy realm was hampered by the scarcity of adequate techniques for handling strongly coupled Yang-Mills theories analytically. The discovery of the AdS/CFT correspondence [1, 2] has given promise for this situation to improve in a qualitative way. Indeed, the ensuing dualities explicitly relate gauge theories at strong coupling to physically equivalent string theories in ten-dimensional spacetimes which become tractable at least in the weak (string) coupling and curvature limits. These dualities are manifestations of the holographic principle [3] and have triggered an entirely new way of thinking about nonperturbative QCD.

The currently best understood dualities deal with supersymmetric and conformal gauge theories, however, and so far the existence of an exact QCD dual and its explicit form have

not been established *ab initio*. The pioneering applications to QCD and hadron physics¹ therefore relied on a minimal infrared (IR) deformation of the anti-de Sitter (AdS) metric to model confinement [5]. By restricting the fifth dimension to a compact interval, this “hard IR wall” geometry softly breaks conformal symmetry in a way consistent with high-energy QCD phenomenology (including counting rules for exclusive scattering amplitudes etc.) [5]. The extension of this minimal approach into a search program for the holographic dual of QCD, guided by experimental information from the gauge theory side, is often referred to as AdS/QCD.

At present, this program is being pursued along two complementary lines. The first one is mostly bottom-up: it assumes a five-dimensional, local effective field theory in IR-deformed AdS₅ spacetime (and potentially in additional background fields of stringy origin) to describe the gravity dual of QCD and attempts to constrain its form and parameters by experimental information. For various works in this direction see [6–12] and references therein. The main virtue of this approach lies in gathering and organizing information on holographic QCD by using the wealth of accurate experimental data available on the gauge theory side. The second type of approach is more directly guided by the underlying, ten-dimensional brane anatomy of the gravity dual and attempts to maintain closer ties to it.² Up to now, however, this typically comes at the price of less direct relations to QCD. For recent work along these lines see for example [14–16] and references therein.

Over the last years AdS/QCD has met with considerable success in describing hadron properties, the heavy quark potential [17–19], vacuum condensates [20], QCD scattering amplitudes at high energy [5, 15] etc. Many of these results were obtained on the basis of the minimal hard wall implementation of IR effects. Although very useful in several respects, the hard wall is also an oversimplification and has revealed shortcomings when more complex and quantitative QCD properties are considered. The perhaps most important limitation discovered so far is that it predicts quadratic instead of linear square-mass trajectories both as a function of spin and radial excitation quantum numbers [7, 10, 21], in contrast to experimental data and semiclassical string model arguments (for highly excited states) which relate linear trajectories to linear quark confinement [22].

Different ways to overcome this problem in the meson sector have recently been proposed in refs. [11, 19, 23]. As a result, linear Regge trajectories $M^2 \propto J$ for spin- J excitations or the analogous radial excitation trajectories $M^2 \propto N$ of mesons could be reproduced by different holographic models. Although the baryon sector [7, 24, 25] exhibits similarly pronounced empirical trajectories of the above type [26], however, a dual description for them has not yet been found. Our primary goals in the present paper will therefore be to understand how linear baryon trajectories can arise in the AdS/QCD framework, and to establish dual descriptions for additional spectral signatures of linear confinement.

To this end, we will focus on another striking and systematic feature in the light hadron

¹Applications to remoter “cousins” of QCD, and in particular to their glueball spectrum, have a longer history. See for example refs. [4].

²At present there is not enough calculational control over string theory in curved spacetimes to allow for a strict top-down approach. Nevertheless, on the maximally symmetric AdS₅ × S⁵ a complete solution of the world-sheet string theory may ultimately become feasible even in the high-curvature regime [13].

spectrum, namely the combined linear square-mass trajectories

$$M^2 = M_0^2 + W(N + L) \tag{1.1}$$

of Regge type on which radial (N) and orbital angular momentum (L) excitations join. The trajectory structure (1.1) is experimentally well established for both the light meson and baryon resonances. Two fits to the meson data yield mutually consistent mean slopes $W = 1.25 \pm 0.15 \text{ GeV}^2$ [27] and $1.14 \pm 0.013 \text{ GeV}^2$ [28], respectively. The fit to the light baryon resonances (i.e. to those consisting of up, down and strange quarks) results in the somewhat smaller but still compatible slope $W = 1.081 \pm 0.035 \text{ GeV}^2$ [26]. Hence the value $W \sim 1.1 \text{ GeV}^2$ is approximately universal³ for all trajectories [29]. The ground state masses M_0 , on the other hand, are channel dependent.

Our work will rely on the dual representation of hadronic states with higher intrinsic angular momenta by metric fluctuations [7, 16, 21] which suggests itself for our purposes because it provides direct access to orbital excitations. The main strategy will be to devise and apply a method for deriving IR deformed gravity duals which incorporate the experimental information contained in the hadron trajectories (1.1). After a brief summary of pertinent facts and results from AdS/QCD and the hard-wall metric in section 2, we set out heuristically in section 3 by determining a minimal modification of the AdS mode dynamics which generates the linear trajectories (1.1) in both meson and baryon spectra. Subsequently, we explicitly construct the IR deformations of AdS₅ which encode the same dynamics, by deriving and solving differential equations for the conformal symmetry breaking part of the warp factor in section 4. Several new features of the resulting gravity background are discussed in section 5. In section 6 we determine the value of the conformal breaking scale and compare the results of our holographic description to experimental data. In section 7, finally, we conclude with a summary of our findings and mention a few avenues for future improvements and applications.

2. AdS/CFT correspondence and hadron spectrum

The gauge/string duality [1, 2] maps type IIB string theories in curved, ten-dimensional spacetimes into gauges theories which live on the (flat) 3+1 dimensional boundaries. For an UV-conformal gauge theory like QCD, the dual string spacetime is the product of a five-dimensional non-compact part which asymptotically (i.e. close to the boundary) approaches the anti-de Sitter space AdS₅ (R) of curvature radius R , and a five-dimensional compact Einstein space X_5 (where $X_5 = S^5(R)$ for the maximally supersymmetric gauge theory) with the same intrinsic size scale. The line element therefore takes the form [5]

$$ds^2 = e^{2A(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) + R^2 ds_{X_5}^2 \tag{2.1}$$

(in conformal Poincaré coordinates) where $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric. Since $A \neq 0$ breaks conformal invariance explicitly, one has to require $A(z) \rightarrow 0$ as $z \rightarrow 0$

³This may be an indication for the conformal symmetry breaking scale $\propto \Lambda_{\text{QCD}}$ to be approximately hadron independent in the light flavor sector, as noted for orbital excitations in ref. [7].

in order to reproduce the conformal behavior of asymptotically free gauge theories at high energies. The string modes⁴ $\phi_i(x, z) = e^{-iP_i x} f_i(z)$ dual to physical states of the gauge theory are particular solutions of the wave equations⁵ in the geometry (2.1) and fluctuations around it, and potentially in additional background fields of stringy origin [2].

Casting these wave equations into the equivalent form of Sturm-Liouville type eigenvalue problems, one finds

$$[-\partial_z^2 + V_M(z)] \varphi(z) = M_M^2 \varphi(z) \tag{2.2}$$

for the normalizable string modes $\varphi(z) = g(z) f_M(z)$ dual to spin-0 ($M = S$) and spin-1 ($M = V$) mesons as well as from the iterated Dirac and Rarita-Schwinger equations

$$[-\partial_z^2 + V_{B,\pm}(z)] \psi_{\pm}(z) = M_B^2 \psi_{\pm}(z) \tag{2.3}$$

for the string modes $\psi_{\pm}(z) = h(z) f_{B,\pm}(z)$ dual to spin-1/2 and 3/2 baryons (where \pm denote the two chiralities of the fermions with $i\gamma^5 \psi_{\pm} = \pm \psi_{\pm}$) [7]. The potentials $V_{M,B}$ contain all relevant information on the string mode masses and the background metric (2.1) which also determines the functions $g(z)$, $h(z)$ introduced above. The eigenvalues $M_{M,B}^2$ constitute the mass spectrum of the four-dimensional gauge theory on the AdS boundary. The boundary conditions for the eigensolutions φ and ψ_{\pm} are supplied by specifying the corresponding gauge theory operator according to the AdS/CFT correspondence [2] (cf. eq. (2.4)) and by the requirement of normalizability (and minimal string action in case of ambiguities) of the eigenmodes. In some cases a further, less well determined boundary condition is imposed in the infrared, at $z = z_m$, in order to break conformal symmetry.

The AdS/CFT correspondence establishes the link between the string mode solutions of eqs. (2.2), (2.3) and physical states on the gauge theory side by prescribing an UV (i.e. $z \rightarrow 0$) boundary condition for the solutions $f_i(z)$ of the five-dimensional field equations [2]. More specifically, for the dual of states $|i\rangle$ with four-dimensional spin 0 one has to select the solution which behaves as $f_i(z) \xrightarrow{z \rightarrow 0} z^{\Delta_i}$ where Δ_i is the conformal dimension of the lowest-dimensional gauge theory operator which creates the state $|i\rangle$. The wave functions of states with four-dimensional spin σ_i acquire an extra boost factor $z^{-\sigma_i}$, so that the boundary condition generalizes to

$$f_i(z) \xrightarrow{z \rightarrow 0} z^{\tau_i}, \quad \tau_i = \Delta_i - \sigma_i \tag{2.4}$$

where the scaling dimension Δ_i of the gauge-invariant operator is replaced by its twist τ_i [5]. The lightest string modes are then associated with the leading twist operators, and therefore with the valence quark content of the low-spin (i.e. spin 0, 1/2, 1, and 3/2) hadron states [7, 21]. The duals of their orbital excitations (which have no counterparts in the supergravity spectra) and hence of higher-spin hadrons are identified with fluctuations about the AdS background [7, 16].

⁴The dependence on the four dimensions x and on the fifth dimension z factorizes at least in the asymptotic AdS region.

⁵The Klein-Gordon, Dirac and Rarita-Schwinger equations on AdS₅ are discussed e.g. in refs. [30, 31].

The leading-twist interpolators contain the minimal number of quark fields q necessary to determine the valence Fock states of the hadrons. Their intrinsic orbital angular momentum L is created by symmetrized (traceless) products of covariant derivatives D_ℓ . This results in the operators $\mathcal{O}_{M,\bar{\tau}=L+2} = \bar{q}\Gamma D_{\{\ell_1 \dots \ell_m\}}q$ with $\Gamma = 1, \gamma^5, \gamma^\mu$ for scalar, pseudoscalar and vector mesons, and $\mathcal{O}_{B,\bar{\tau}=L+3} = qD_{\{\ell_1 \dots \ell_q\}}qD_{\ell_{q+1}} \dots D_{\ell_m\}}q$ corresponding to spin-1/2 (or 3/2) baryons, with $L = \sum_i \ell_i$. The boundary condition (2.4) is imposed on the solutions by setting the values of the five-dimensional masses $m_{5,H}$ (which determine the small- z behavior) according to the twist dimension $\bar{\tau}$ of the hadron interpolator to which they are dual [2, 5, 21], i.e.

$$m_{5,H}R \rightarrow \begin{cases} \sqrt{\bar{\tau}(\bar{\tau} - d)} & \text{for spin-0 mesons,} \\ \sqrt{\bar{\tau}(\bar{\tau} - d) + d - 1} & \text{for vector mesons,} \\ \bar{\tau} - 2 & \text{for baryons,} \end{cases} \quad (2.5)$$

where d is the dimension of the boundary spacetime. The twist dimension of the interpolating operators thereby enters the field equations and the potentials $V_{M,B}$.

As outlined above, the duality of string modes to hadrons is based on their association with the lowest-dimensional, gauge-invariant QCD interpolators of matching quantum numbers. Hence this identification is incomplete as long as fundamental quark flavor and chiral symmetry are not properly accounted for. In the gravity dual, fundamental flavor arises from open string sectors with the strings ending on added D-brane stacks [32]. This mechanism has an effective bulk description in terms of chiral gauge symmetries [8, 9] whose implementation into our framework will be left to future work.

Nevertheless, a few comments on these issues and their impact especially on the baryon sector may be useful already at the present stage. The simplest and currently most popular top-down approach to quark flavor in the meson sector introduces a D7-brane stack into (potentially deformed) AdS, with open strings stretching between the D7- and D3-branes and with the duals of flavored mesons living on their intersection [32–34]. Baryons require an additional D5-brane (wrapped around a compact part of the ten-dimensional space) which contains baryonic vertices [35] whose attached strings pick up flavor by ending on the D7-branes as well [33].

This construction is difficult to handle explicitly, however, and the simpler propagation of fermionic fluctuations on the D3/D7 intersection *without* the D5-brane was therefore recently studied as a preparatory step [31]. The resulting modes are dual to the superpartners of the mesons investigated in ref. [33], i.e. they correspond to fermionic bound states of fundamental scalars (squarks) and fundamental as well as adjoint spinors. These modes are solutions of Dirac equations similar to the one used in our framework but fall into degenerate ($\mathcal{N} = 2$) supermultiplets with their mesonic partners.

Although bottom-up duals of baryons (with the correct $O(N_c)$ scaling behavior of their masses) obey Dirac equations as well [31, 36], they differ from the above “mesinos” in their assignment to gauge-theory operators. Consequently they are subject to other AdS/CFT boundary conditions and generate different mass spectra, as expected on physical grounds. Such bottom-up descriptions of baryon duals were considered in ref. [36], on which we partially rely here, and also integrated [25] into the approach of ref. [8].

Another interesting dual representation of baryons has recently emerged from a D4/D8-brane construction for fundamental flavor and chiral symmetry [24]. This approach generates a Chern-Simons gauge theory in the bulk whose instantons are dual to solitonic baryons of Skyrme type [37], i.e. to collective excitations of the meson fields which carry topological baryon number and have masses of $O(N_c)$ as well. Since low-energy properties of Skymions can be described by Dirac fields (as they appear e.g. in chiral perturbation theory [38]), this approach to baryon duals may in fact be complimentary to those of refs. [25, 31, 36] outlined above.

The conformal potentials induced by the pure AdS₅ metric⁶ (i.e. by eq. (2.1) with $A \equiv 0$) are proportional to $1/z^2$:

$$V_M^{(\text{AdS})}(z) = \left[\frac{15}{4} - (d-1)\delta_{M,V} + m_{5,M}^2 R^2 \right] \frac{1}{z^2}, \quad (2.6)$$

$$V_{B,\pm}^{(\text{AdS})}(z) = m_{5,B} R [m_{5,B} R \mp 1] \frac{1}{z^2}. \quad (2.7)$$

Hence the normalizable eigensolutions of eqs. (2.2) and (2.3) are Bessel functions whose order and eigenvalues depend on the boundary conditions for the solutions $f_i(z)$ of the field equations (cf. ref. [7]). The conformal invariance inherited from the AdS metric, however, prevents these potentials from carrying direct information on IR effects of QCD.

The simplest way to approximately implement such IR effects, and in particular confinement, is to impose a Dirichlet boundary condition on the string modes at a finite IR scale z_m . This approach is prevalent among current bottom-up models and amounts to a sudden onset of conformal symmetry breaking by a “hard-wall” horizon of the metric at the IR brane [5], i.e.

$$\begin{aligned} e^{2A_{\text{hw}}(z)} &= \theta(z_m - z), \\ z_m &= \Lambda_{\text{QCD}}^{-1}, \end{aligned} \quad (2.8)$$

and reduces the five-dimensional, noncompact space to an AdS₅ slice. Even this minimal implementation of non-conformal IR effects into approximate QCD duals (with only one free parameter related to Λ_{QCD}) can already predict a remarkable amount of hadron physics, as outlined in the introduction. The investigation of hadron spectra and wave functions in the approach of refs. [7, 21], in particular, gave a good overall account of the angular momentum excitation spectra for both mesons and baryons.

In view of its simplicity, however, it is not surprising that the hard wall confinement also reveals shortcomings. In particular, it predicts the square masses of radially and orbitally excited hadrons to grow quadratically with N and L [7, 10], in conflict with the

⁶The modes dual to baryons originate from the ten-dimensional Dirac equation. Hence the nonvanishing eigenvalue of the lowest-lying Kaluza-Klein (KK) mode of the Dirac operator on the compact space X_5 [39] adds to the five-dimensional mode mass $m_{5,B}$ (cf. e.g. ref. [7]). Since the AdS/CFT boundary conditions replace the whole mass term by a function of the twist dimension of the gauge theory operator to be sourced, however, the KK eigenvalue will not appear explicitly in the final expressions and has already been absorbed into $m_{5,B}$.

linear Regge-type trajectories found experimentally⁷ and expected from the semiclassical treatment of simple, relativistic string models [22]. While more detailed implementations of conformal symmetry breaking were able to resolve this problem in the meson sector [11, 19, 23], linear baryon trajectories have so far not been obtained from a gravity dual. As mentioned in the introduction, this provides part of our motivation to search for a holographic representation which reproduces linear trajectories in the baryon sector as well. (Note, incidentally, that the approach of ref. [11], at least in its simplest form where a dilaton $\Phi(z) \propto z^2$ is solely responsible for conformal symmetry breaking, will not lead to linear trajectories in the baryon sector since the dilaton interaction can be factored out of the Dirac equation and hence does not affect the baryon spectrum.)

3. Linear trajectories of radially and orbitally excited hadrons from AdS type potentials

We are now going to develop a gravity dual which manifests soft conformal symmetry breaking directly in the potentials and is capable of generating linear trajectories in both meson and baryon spectra. To this end, we find in the present section suitable potentials heuristically and show that they indeed reproduce the trajectories (1.1). In the subsequent section 4 we then construct the IR deformations of the metric (2.1) which encode them holographically.

A natural guess for the z dependence of potentials $V_{M,B}^{(LT)}$ which are able to generate the linear trajectorial (LT) structure (1.1) is that it should be of oscillator type in the infrared (i.e. quadratically rising with z for $z \rightarrow \infty$). The more challenging question is how to realize this behavior in a universal way, i.e. on the basis of just one *a priori* free mass scale λ and such that the same slope W and $N + L$ dependence emerges in both meson and baryon channels. It turns out that this can be achieved at the level of the twist dimensions which enter the five-dimensional mass terms according to eq. (2.5) after imposing the AdS/CFT boundary conditions (2.4). Indeed, all the necessary information on conformal symmetry breaking can be implemented into the AdS₅ potentials (2.6), (2.7) by replacing

$$\bar{\tau}_i \rightarrow \bar{\tau}_i + \lambda^2 z^2. \tag{3.1}$$

(The hard wall restriction (2.8) of the AdS space becomes obsolete.) The heuristic rule (3.1) implies that the product of the five-dimensional masses m_i and the square root $a(z) = R/z$ of the AdS warp factor grow linearly with z for $z \rightarrow \infty$, thus foreshadowing the linear trajectories (1.1) for both mesons and baryons. The role of the hadron-independent mass scale λ will become more explicit below and in section 6 where we relate it to the trajectory slope W and to the QCD scale.

As expected from *soft* conformal symmetry breaking, the replacement (3.1) does affect neither the $z \rightarrow 0$ behavior of the field equations nor that of their solutions. Both the

⁷In the hard wall model the first radial excitations of light mesons and the nucleon, identified by a node in the string mode, appear at masses of about 1.8 GeV and 1.85 GeV, respectively [7], and are therefore difficult to reconcile with the experimental $\pi(1300)$, $\rho(1450)$ and Roper $N(1440)P_{11}$ resonances. These shortcomings are suspected to be artifacts of the hard wall metric as well [7].

conformal symmetry on the UV brane and the $z \rightarrow 0$ boundary conditions (2.4) from the AdS/CFT dictionary are therefore preserved. As a consequence of the above procedure, the mass terms in the pure AdS₅ potentials (2.6), (2.7) carry all information not only on the twist dimension (and thus orbital excitation level) of the dual QCD operators but also on the deviations from conformal behavior in the infrared. The underlying physical picture will be discussed in section 5.

Recalling the expression $\bar{\tau}_M = L + 2$ for the twist dimension of the meson interpolators from section 2 and making use of the replacement (3.1) then turns the mesonic AdS potential (2.6) into

$$V_M^{(\text{LT})}(z) = \left[(\lambda^2 z^2 + L)^2 - \frac{1}{4} \right] \frac{1}{z^2} \quad (3.2)$$

(which holds for both spin 0 and 1) while the AdS potential (2.7), associated with the baryon interpolator of twist dimension $\bar{\tau}_B = L + 3$, becomes

$$V_{B,\pm}^{(\text{LT})}(z) = \{ (L+1)(L+1 \mp 1) + [2(L+1) \pm 1] \lambda^2 z^2 + \lambda^4 z^4 \} \frac{1}{z^2}. \quad (3.3)$$

The normalizable solutions of the corresponding eigenvalue problems (2.2) and (2.3) can be found analytically. For the mesons one obtains

$$\varphi_{N,L}(z) = \mathcal{N}_{M;L,N} (\lambda z)^{L+1/2} e^{-\lambda^2 z^2/2} \mathbf{L}_N^{(L)}(\lambda^2 z^2) \quad (3.4)$$

where the $\mathbf{L}_N^{(\alpha)}$ are generalized Laguerre polynomials [40] and $\mathcal{N}_{H;L,N}$ are normalization constants. For the (spin 1/2 and 3/2) baryons one similarly finds

$$\psi_{N,L,+}(z) = \mathcal{N}_{B+;L,N} (\lambda z)^{L+1} e^{-\lambda^2 z^2/2} \mathbf{L}_N^{(L+1/2)}(\lambda^2 z^2), \quad (3.5)$$

$$\psi_{N,L,-}(z) = \mathcal{N}_{B-;L,N} (\lambda z)^{L+2} e^{-\lambda^2 z^2/2} \mathbf{L}_N^{(L+3/2)}(\lambda^2 z^2). \quad (3.6)$$

Note that all eigenfunctions have appreciable support only over short distances, in the small region $z \lesssim \sqrt{2}\lambda^{-1} \simeq \Lambda_{\text{QCD}}^{-1}$ (cf. section 6) close to the UV brane, which is an expected consequence of confinement.

The corresponding eigenvalues

$$M_M^2 = 4\lambda^2 \left(N + L + \frac{1}{2} \right), \quad (3.7)$$

$$M_B^2 = 4\lambda^2 \left(N + L + \frac{3}{2} \right) \quad (3.8)$$

show that the square masses of both mesons and baryons are indeed organized into the observed $N + L$ trajectories. Moreover, the spectra (3.7) and (3.8) predict the universal slope

$$W = 4\lambda^2 \quad (3.9)$$

for both meson and baryon trajectories in terms of the IR scale λ . They also exhibit a mass gap (of order \sqrt{W}), another hallmark of confining gauge theories, and the intercepts

$M_{i,0}^2$ (cf. eq. (1.1)) relate the slope of the trajectories in a new way to their ground state masses,

$$M_{M,0}^2 = \frac{W}{2}, \tag{3.10}$$

$$M_{B,0}^2 = \frac{3W}{2}. \tag{3.11}$$

The quantitative implications of these relations will be discussed in section 6.

4. Derivation of the equivalent IR deformations of AdS₅

Although the existence of potentials (3.2) and (3.3) which generate linear trajectories of the type (1.1) is encouraging, we have not yet provided any dynamical justification for them. Indeed, for the spectra (3.7), (3.8) to be the outcome of a dual gauge theory, and for the hadronic quantum numbers to be associated with the correct interpolating operators, one has to show that they emerge from stringy fluctuations in a bulk gravity background. In the present section we are going to establish this missing link by constructing the corresponding background metric explicitly.

Of course, *a priori* the existence of such a bulk geometry is far from guaranteed, given the quasi *ad-hoc* nature of the heuristic rule (3.1) which we used to find the potentials in the first place. Moreover, it will prove sufficient to consider just the minimal set of background fields,⁸ consisting of the metric only. In fact, we will show that even the simplest type of IR modifications of the AdS₅ metric, due to a non-conformal warp factor $e^{2A(z)}$ as anticipated in eq. (2.1), can generate the potentials (3.2), (3.3). The success of this minimal approach can be at least partially understood by noting that the potentials contain effects of an order of magnitude which should arise from leading-order contributions to the effective gravity action, i.e. from the metric. Higher-order contributions due to dimensionful background fields (as e.g. the dilaton), in contrast, would be suppressed by potentially large mass scales.

In order to prove the above assertions and to construct the non-conformal warp factor, we first obtain the five-dimensional field equations for string modes of the form $\phi(x, z) = f_S(z) e^{-iPx}$ (spin 0) and $V_z(x, z) = f_V(z) \varepsilon_z e^{-iPx}$ (spin 1) which are dual to mesons and propagate in the background of the metric (2.1) with an *a priori* unspecified warp function $A(z)$. The ensuing bulk equations for the z dependent part of the string modes are

$$\left[\partial_z^2 + 3 \left(A' - \frac{1}{z} \right) \partial_z - \left(m_{5,S} R \frac{e^A}{z} \right)^2 + M^2 \right] f_S(z) = 0 \tag{4.1}$$

with $A' \equiv \partial_z A$ and the four-dimensional invariant square mass $M^2 = P^2$, as well as

$$\left[\partial_z^2 + 3 \left(A' - \frac{1}{z} \right) \partial_z + 3 \left(A'' + \frac{1}{z^2} \right) - \left(m_{5,V} R \frac{e^A}{z} \right)^2 + M^2 \right] f_V(z) = 0. \tag{4.2}$$

⁸An effective z dependence of the dual string mode masses may of course also arise from additional background fields, as for example from a Yukawa-coupled Higgs field [41].

The string modes dual to baryons can be decomposed into left- and right-handed components,

$$\Psi(x, z) = \left[\frac{1 + \gamma^5}{2} f_+(z) + \frac{1 - \gamma^5}{2} f_-(z) \right] \Psi_{(4)}(x), \quad (4.3)$$

where $\Psi_{(4)}(x)$ satisfies the Dirac equation $(i\gamma^\mu \partial_\mu - M)\Psi_{(4)}(x) = 0$ on the four-dimensional (Minkowski) boundary spacetime. As a consequence, the iterated five-dimensional Dirac equation for Ψ reduces to

$$\left[\partial_z^2 + 4 \left(A' - \frac{1}{z} \right) \partial_z + 2 \left(A'' + \frac{1}{z^2} \right) + 4 \left(A' - \frac{1}{z} \right)^2 - \left(m_{5,B} R \frac{e^A}{z} \right)^2 \mp m_{5,B} R \frac{e^A}{z} \left(A' - \frac{1}{z} \right) + M^2 \right] f_\pm(z) = 0 \quad (4.4)$$

for the right and left handed modes with chiralities $i\gamma^5 f_\pm = \pm f_\pm$.

These equations can be translated into the equivalent form of Schrödinger-type eigenvalue problems (2.2) and (2.3) for $\varphi(z)$ and $\psi_\pm(z)$ by writing $f_{S,V}(z) = (\lambda z e^{-A})^{3/2} \varphi(z)$ and $f_\pm(z) = (\lambda z e^{-A})^2 \psi_\pm(z)$ which eliminates the first-derivative terms. The corresponding generalizations of the AdS₅ potentials (2.6), (2.7) are then read off from the eigenvalue equations as

$$V_S(z) = \frac{3}{2} \left[A'' + \frac{3}{2} A'^2 - 3 \frac{A'}{z} + \frac{5}{2} \frac{1}{z^2} \right] + m_{5,S}^2 R^2 \frac{e^{2A}}{z^2}, \quad (4.5)$$

$$V_V(z) = \frac{3}{2} \left[-A'' + \frac{3}{2} A'^2 - 3 \frac{A'}{z} + \frac{1}{2} \frac{1}{z^2} \right] + m_{5,V}^2 R^2 \frac{e^{2A}}{z^2} \quad (4.6)$$

and

$$V_{B,\pm}(z) = m_{5,B} R \frac{e^A}{z} \left[\pm \left(A' - \frac{1}{z} \right) + m_{5,B} R \frac{e^A}{z} \right]. \quad (4.7)$$

The pure AdS₅ potentials are contained in these expressions for $A \equiv 0$. The AdS/CFT boundary condition, which relates the eigensolutions to the dual meson (baryon) operators of twist dimension $\bar{\tau}_M = L + 2$ ($\bar{\tau}_B = L + 3$), is imposed by adjusting the mass terms,

$$m_{5,S}^2 R^2 = \bar{\tau}_M(\bar{\tau}_M - 4) = L^2 - 4, \quad (4.8)$$

$$m_{5,V}^2 R^2 = \bar{\tau}_M(\bar{\tau}_M - 4) + 3 = L^2 - 1, \quad (4.9)$$

$$m_{5,B} R = \bar{\tau}_B - 2 = L + 1, \quad (4.10)$$

as outlined above. Equating the general potentials (4.5) - (4.7) to their heuristic counterparts (3.2), (3.3) leaves us with differential equations for the corresponding warp functions. Their solutions (subject to appropriate boundary conditions), finally, determine the equivalent background metric of the form (2.1).

As already mentioned, it is *a priori* uncertain whether there exists an approximate gravity dual whose IR deformation can reproduce a given five-dimensional potential and spectrum, simply because it may not result from a boundary gauge theory. In the above

approach this is reflected in the fact that the nonlinear, inhomogeneous differential equations for $A(z)$ may not have physically acceptable solutions. Our next task is therefore to construct and analyze the solution spaces of these differential equations for the heuristic potentials (3.2), (3.3).

4.1 Baryon sector

We begin our discussion in the baryon sector where the equation for A is of first order and hence has less and generally simpler solutions. Equating the potential (3.3), whose representation in terms of the background geometry we wish to construct, to the potential (4.7) for general A results in the nonlinear, inhomogeneous differential equation

$$\pm (zA' - 1) + le^A - [l(l \mp 1) + (2l \pm 1)\lambda^2 z^2 + \lambda^4 z^4] (le^A)^{-1} = 0 \quad (4.11)$$

of first order (where $l \equiv L + 1$ and \pm refers to the two baryon chiralities) whose solution determines the non-conformal part $\exp[2A_B(z)]$ of the equivalent warp factor.

Remarkably, the exact (and essentially unique) solution of eq. (4.11) subject to the conformal boundary condition $A_B(0) = 0$ can be found analytically and turns out to be

$$A_B(z) = \ln \left(1 + \frac{\lambda^2 z^2}{L+1} \right). \quad (4.12)$$

Note that the same solution holds for both baryon chiralities. The leading contribution to the non-conformal part of the warp factor at small $z^2 \ll \lambda^{-2}$ is therefore

$$e^{2A_B} = e^{\frac{2}{L+1}\lambda^2 z^2 + O(\lambda^4 z^4)} \quad (4.13)$$

which has the form of the analogous warp factor $\exp(cz^2/2)$ used in refs. [19, 42] (together with a constant dilaton) to obtain a linear quark potential and a linear (mesonic) Regge trajectory.

4.2 Scalar meson sector

In the following we are going through the analogous construction for A_S in the spin-0 meson sector, which will turn out to be more multi-faceted. Equating the meson potential (3.2) to its general- A counterpart (4.5) produces again a nonlinear, inhomogeneous equation, but for A_S it is of second order:

$$z^2 A'' + \frac{3}{2} (zA')^2 - 3zA' + \frac{2}{3} (L^2 - 4) (e^{2A} - 1) - \frac{2}{3} \lambda^2 z^2 (\lambda^2 z^2 + 2L) = 0. \quad (4.14)$$

In addition to $A_S(0) = 0$, its solutions require a second boundary condition which as of yet remains unspecified and will be determined below. This added freedom provides one of the reasons for the solution space in the meson sector to be larger and more diverse than in the baryon sector.

In addition, the L dependence of the solutions A_S is more heterogeneous since the sign of the mass term $m_{5,S}^2 R^2$ in the field equation can be either negative, zero, or positive (cf. eq. (4.8)). These three cases generate qualitatively different solution behaviors. The

positive sign corresponds to $L > 2$ and is associated with irrelevant gauge theory operators according to the renormalization group (RG) classification. The solutions for $L > 2$ will turn out to be qualitatively similar to those in the baryon sector. The massless case $m_{5,S}^2 R^2 = 0$ corresponds to $L = 2$ and to a marginal operator in the RG sense. The duals of the lowest orbital excitations $L = 0, 1$, finally, represent relevant operators and are tachyons⁹ with $m_{5,S}^2 R^2 = -4, -3$. In the following, we will discuss these three cases in turn.

$L = 0, 1$: due to their tachyonic nature, the $L = 0, 1$ solutions are perhaps the most interesting ones. The negative mass term together with the specific inhomogeneity generated by the potential (3.2) forces these solutions to develop a singularity at finite $z = z_m$, which restricts the spacetime to an AdS₅ slice.¹⁰ The position and sign of these singularities depends on the second boundary condition for $A_S(z)$. At $z = 0$ this boundary condition may e.g. be imposed¹¹ on $A_S'''(0)$ for $L \neq 0$ and on $A_S''''(0)$ for $L = 0$. The singularities have positive (negative) sign, i.e. $A \rightarrow \pm\infty$, if $A_S'''(0)$ (or $A_S''''(0)$ for $L = 0$) is chosen larger (smaller) than a critical value.

Independently of the sign of the singularities, furthermore, the warp factor

$$\frac{R^2}{z^2} e^{2A_S(z)} \geq c(z_m) \geq 0 \tag{4.15}$$

($c(z_m) = R^2 \exp[2A_S(z_m)]/z_m^2 = 0$ for negative singularities) of all solutions remains bounded from below for all z up to z_m where the dual spacetime ends. The above behavior provides a sufficient confinement criterion in the five-dimensional holographic description. The Wilson loop [17] then shows an area law (since the strings are localized at z_m) and the gauge theory develops the expected mass gap $M_{\min} \geq z_m^{-1}$ [18]. In fact, the hard-wall horizon (2.8) may be considered as a simple model for this type of behavior. It corresponds to the development of an abrupt negative singularity of A at z_m .

The origin and locus of the negative singularities can be understood quantitatively by obtaining a series solution for $z \ll \sqrt{2}\lambda^{-1}$ in the form

$$A_S(z) = \frac{1}{2} \ln \left[1 + \sum_{n=1}^{\infty} A_n(L) \left(\frac{\lambda^2 z^2}{2} \right)^n \right] \tag{4.16}$$

which already incorporates both boundary conditions, i.e. $A(0) = 0$ and a second one to become explicit below. The coefficients A_n may be calculated by inserting the expan-

⁹Scalar AdS₅ tachyons with masses satisfying the Breitenlohner-Freedman bound $m^2 R^2 \geq -d^2/4$ (which includes the cases we encounter here for $d = 4$) do not cause instabilities, as has been known for some time [43].

¹⁰In our case this is a geometric consequence of requiring the holographic dual to generate potentials which exhibit the linear spectral trajectories (3.7), (3.8). Alternatively, additional branes may restrict the fifth dimension in the IR [44].

¹¹The analysis of the linearized approximation to eq. (4.14) in appendix A shows that $A_S(0) = 0$ automatically implies $A_S'(0) = 0$, $A_S''(0) = 2L\lambda^2/(L^2 - 7)$ for $L \neq 0$ as well as $A_S'(0) = A_S''(0) = A_S'''(0) = 0$, $A_S''''(0) = -3\lambda^4$ for $L = 0$, as a consequence of the inhomogeneity.

sion (4.16) into eq. (4.14). The first three are (for $L \neq 2$)

$$A_1(L) = \frac{4L}{L^2 - 7}, \quad A_2(L) = \frac{4}{L^2 - 4} \left(1 - \frac{9}{4} A_1^2 \right), \quad A_3(L) = \frac{-12}{L^2 + 5} A_1^3. \quad (4.17)$$

In general, the inhomogeneity of eq. (4.14) determines the leading small- z behavior of the A_S , as revealed by the solutions (A.2) - (A.4) of the linearized equation given in appendix A. For $L = 0, 1$, in particular, it forces the z dependence inside the logarithm to start out quadratically and yields

$$A_{S,L=0,1}(z) = \frac{1}{2} \ln \left[1 + A_1 \frac{\lambda^2 z^2}{2} + \dots + O(\lambda^6 z^6) \right] \\ \xrightarrow{z^2 \ll 2\lambda^{-2}} \frac{L}{L^2 - 7} \lambda^2 z^2 + \frac{1}{16} (2A_2 - A_1^2) \lambda^4 z^4 + O(\lambda^6 z^6). \quad (4.18)$$

(The second boundary condition can be read off from this expression.) Equation (4.18) also implies that the solutions with $L = 0, 1$ turn negative for $z \gtrsim 0$. At large z , on the other hand, the inhomogeneity rises $\propto z^4$ and demands the modulus of A_S to grow as well. At some finite z_m the nonlinearity $\propto (e^{2A} - 1)$ in eq. (4.14) will therefore become too negative (for $A_S > 0$ and $L < 2$) to be counterbalanced by the derivative terms. To avoid conflict with the increasingly positive inhomogeneity, the solution then develops a singularity at z_m . As already mentioned, the sign of the singularity depends on the second boundary condition. A negative singularity occurs if the slope of A_S near $z = 0$ is smaller than a critical value such that the argument of the logarithm in eq. (4.16) eventually reaches zero. This singularity is quantitatively reproduced by the series solution (4.16) as long as z_m lies inside its range of validity.

If the slope of A_S at small z exceeds the critical value, on the other hand, the zero in the argument of the logarithm is avoided. Instead, the argument stays positive with increasing z , passes through a minimum and starts to increase until it reaches a positive pole singularity at finite z_m . Singularities of this type lie outside the validity range of the expansion (4.16) and imply that the non-conformal part of the warp factor approaches a pole singularity as well. For $L = 0$, e.g., it takes the explicit form

$$e^{2A_S(z)} \sim \frac{12}{\lambda^2 (z - z_m)^2}. \quad (4.19)$$

$L = 2$: for $L = 2$ the mass term in eq. (4.14), and hence the strongest nonlinearity, vanishes. The initial curvature $A''_{S,L=2}(0) = -4\lambda^2/3$ is negative and again set by the inhomogeneity (cf. eq. (A.3)). Consequently, the solution can still develop a negative singularity if its slope close to $z = 0$ remains below a critical value. For larger slopes the solutions turn positive and are nonsingular. Hence $L = 2$ corresponds to the intermediate case in which the solution space contains both singular solutions in which confinement manifests itself by compactifying the fifth dimension (similar to the $L = 0, 1$ cases), and regular solutions analogous to those encountered for $L > 2$ (see below). Of course, the ensuing potential (3.2) is (up to z_m) identical in both cases.

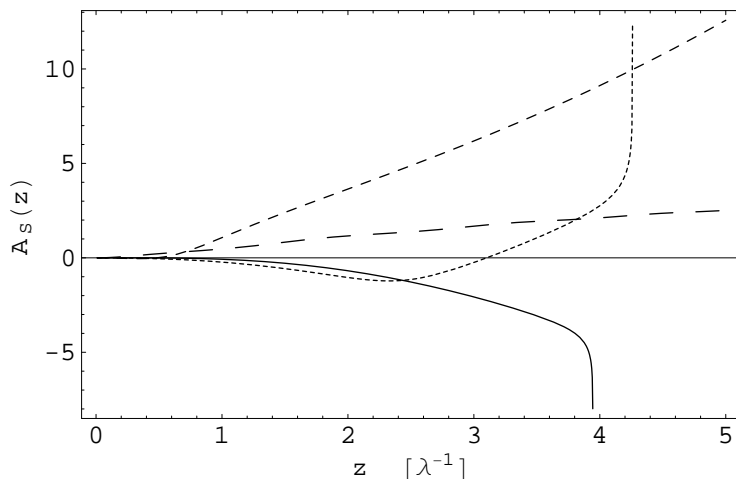


Figure 1: Typical solutions $A_S(z)$ for $L = 0$ (full line, negative sign of singularity selected), $L = 1$ (dotted line, positive sign of singularity selected), $L = 2$ (short-dashed, absence of singularity selected) and $L = 3$ (long-dashed). Note that the dual eigenmodes have significant support only for $z < \sqrt{2}\lambda^{-1}$.

$L > 2$: for all $L > 2$ the main nonlinearity in eq. (4.14) has a positive sign. Moreover, at $z = 0$ the solutions start out with positive curvature $A_S''(0) = 2L\lambda^2 / (L^2 - 7)$ (again dictated by the inhomogeneity, as can be seen from their linearized counterparts (A.4)). In fact, the $A_{S,L>2}$ and the corresponding warp factors remain positive and nonsingular at all z . Typical numerical solutions for A_S with $L = 0, 1, 2$ and 3 are displayed in figure 1.

By construction, any background metric of the form (2.1), with A_S a solution of eq. (4.14), reproduces the potential (3.2) for all existing z . If the background space ends in z direction at z_m , however, the potential (3.2) must end there, too. This does not affect the (low-lying) spectrum as long as $z_m \gg \lambda^{-1}$. Hence for our purpose of maintaining the trajectory (3.7) even when $L = 0, 1$ we choose the second boundary condition such that z_m becomes as large as needed.¹² This selects the singularities of positive sign and implies that the corresponding field modes automatically satisfy Dirichlet (or Neumann) boundary conditions at z_m :

$$f_{S,L=0,1}(z_m) = \left[\lambda z_m e^{-A_S(z_m)} \right]^{3/2} \varphi_{L=0,1}(z_m) = 0. \tag{4.20}$$

Even for radial excitations well beyond those currently experimentally accessible, the $L = 0, 1$ part of the spectrum (3.7) remains therefore unaffected. (Note the Gaussian suppression of the eigenmodes (3.4) for $z_m^2 \gg 2\lambda^{-2}$.) We will elaborate on this issue in section 5.1. (Recall for comparison that the much slower decay of the string mode solutions in pure AdS₅ (Bessel functions) requires that boundary conditions at the hard IR wall have to be imposed by hand. This strongly modifies the spectrum — the masses become proportional to the zeros of Bessel functions [7] — and generates the incorrect $M^2 \propto N^2, L^2$ behavior which is typical for infinite square well or bag potentials.)

¹²We have found numerical solutions of eq. (4.14) with $z_m > 6\lambda^{-1}$ (for $L = 0$).

4.3 Vector meson sector

The equation for the non-conformal warp factor in the spin-1 meson sector,

$$-z^2 A'' + \frac{3}{2} z^2 A'^2 - 3z A' + \frac{2}{3} (L^2 - 1) (e^{2A} - 1) - \frac{2}{3} \lambda^2 z^2 (\lambda^2 z^2 + 2L) = 0, \quad (4.21)$$

is obtained by setting the meson potential (3.2) equal to the general- A potential (4.6). This equation differs from its counterpart (4.14) in the spin-0 sector only by the sign of the A'' term and by the replacement $(L^2 - 4) \rightarrow (L^2 - 1)$ in the coefficient of the main nonlinearity. The inhomogeneity and its L dependence remain identical since they originate from the same bulk potential (3.2) in both cases. Hence the analysis of the solution space is similar to that of section 4.2 and will not be repeated in detail. Instead, we will just highlight qualitative differences in the solution behavior and discuss the numerical solutions of eq. (4.21) and their implications.

The main differences between the solutions of eqs. (4.14) and (4.21) can be rather directly traced to the two differences in the equations mentioned above. The modified coefficient of the exponential term reflects the fact that in the vector meson sector only $L = 0$ corresponds to a tachyonic mode while the $L = 1$ mode is massless and all $L > 1$ modes are massive (cf. eq. (4.9)). This is a consequence of the additional unit of spin carried by the vector mesons. Hence the qualitative changes in the solution behavior occur around $L = 1$ (instead of at $L = 2$). The sign flip of the second-derivative term, furthermore, leads to changes in the sign of the solutions $A_V(z)$ and in the development of singularities. Part of these modifications are determined by the solution behavior at small z which, as in the spin-0 case, can be obtained from the solutions to the linearized version of eq. (4.21) as derived in appendix A.

The above qualitative expectations are corroborated by the numerical solutions (again subject to the conformal boundary condition $A_V(0) = 0$). Their perhaps most important novel feature is that the second boundary condition can be adapted to generate a common qualitative behavior for all L which is singularity-free. More specifically, every regular solution turns negative towards larger z where it decreases monotonically. For $L > 2$ the solutions start out with positive slope at $z = 0$ (cf. eq. (A.8)) which implies that the decrease can set in only after passing through a shallow, positive maximum at finite z . While the $L = 0$ solution (associated with the tachyon mode) stays regular for all choices of the second boundary condition, however, for $L > 0$ one has the additional possibility of solutions which remain positive towards larger z and develop a singularity at $z = z_m$ similar to those encountered in the spin-0 sector for $L = 0, 1$. A set of typical solutions $A_V(z)$ for $L = 0, \dots, 3$ is plotted in figure 2.

To summarize, in the vector meson channel the dual manifestation of confinement in terms of a dynamically compactified fifth dimension is not a necessity but rather an option for the higher orbital excitations (with $L > 1$). In contrast to the scalar sector, furthermore, the underlying singularities are not associated with tachyonic modes.

We conclude this section by noting that the existence of simple gravity duals which reproduce the linear trajectories (1.1), as constructed above, provides additional support for the AdS/QCD program. Moreover, it establishes the basis for calculating gauge the-

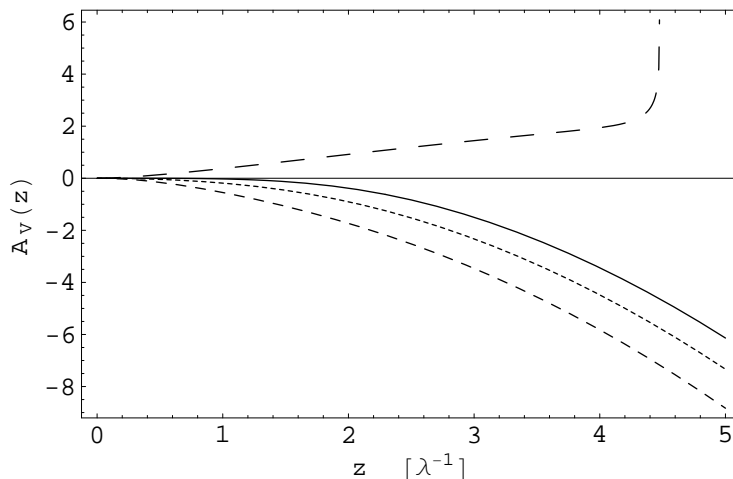


Figure 2: Typical solutions $A_V(z)$ for $L = 0$ (full line), $L = 1$ (dotted line), $L = 2$ (short-dashed, absence of singularity selected) and $L = 3$ (long-dashed, singularity selected). Recall that the dual eigenmodes have significant support only for $z < \sqrt{2}\lambda^{-1}$.

ory correlation functions and observables from the dual mode solutions (3.4)–(3.6) on the basis of the AdS/CFT dictionary [2]. Close to the AdS boundary, all metrics found above have the same qualitative small- z behavior with a non-conformal warp factor of the form $\exp(cz^2)$ (except for scalar mesons with $L = 0$ and vector mesons with $L = 2$), as determined by the inhomogeneous terms in eqs. (4.14), (4.11) and hence directly induced by the underlying bulk potentials. This behavior suggests the formation of a two-dimensional (nonlocal) gluon condensate [45] and indicates its relevance for linear confinement (cf. section 5.4). (For $L = 0$ mesons the standard four-dimensional gluon condensate seems to dominate, on the other hand, which should be compared to the operator product expansion with renormalon-type corrections for the corresponding QCD correlators.) At large L , furthermore, the leading z dependence of the mesonic and baryonic warp factors becomes identical. Finally, an IR cutoff dual to Λ_{QCD} for the fifth dimension emerges as a necessary requirement for weakly orbitally (i.e. $L = 0, 1$) excited scalar mesons and as a possibility for $L > 1$ vector mesons. In our framework this compactification of the fifth dimension occurs dynamically.

5. Discussion of the resulting holographic duals

In the following section we elaborate on several important properties of the IR-deformed gravity backgrounds found above and discuss their physical significance.

5.1 Singularities in the meson sector and linear confinement

In several recent holographic QCD models, the asymptotically free and hence almost conformal region (with at most weakly deformed AdS metric) is assumed to extend down to energies z^{-1} of the order of the QCD scale. It gets broken only in the infrared, by a rather abrupt onset of nonperturbative effects including condensates and confinement.

This scenario is indeed borne out dynamically by our results especially for the lowest (i.e. $L = 0, 1$) orbital spin-0 meson excitations. These are exactly the meson modes accessible in weakly curved dual supergravity backgrounds. As shown above, for them to lie on linear trajectories requires a singular metric of the type [17, 18] emerging in holographic duals of confining theories. In these cases the onset of confinement indeed occurs rather sudden, although not as sudden as in the extreme hard wall case (2.8). We have already traced the origin of these singularities to the tachyonic nature of the corresponding dual modes.¹³ In the vector meson sector, in contrast, singular metrics are possible but not mandatory (depending on the choice of a boundary condition) for orbitally excited resonances, and they are not tachyon-induced.

Physically, it seems that no ($L = 0$) or only a small ($L = 1$) centrifugal barrier allows the corresponding spin-0 meson states to probe different aspects of the IR region and hence to feel a more sudden onset of confinement, in particular at large N . The location of the singularities, however, is set by the inverse mass scale in the second boundary condition and can therefore be put at $z_m \gg \lambda^{-1}$ large enough not to significantly alter the low-lying part of the spectra. Furthermore, semiclassical arguments indicate that highly excited hadrons generally become larger and therefore should be able to explore more of the IR region as well. This may explain why the vector meson metric can become singular for $L > 1$.

The possibility to set $z_m \gg \lambda^{-1}$ implies, in particular, that one could choose extensions of the metric into the region $z \in [z_m - \varepsilon, \infty]$ which yield the same low-lying spectra and wave functions (cf. eq. (4.20)) without any singularities. In the intermediate case of spin-0 mesons with $L = 0$ the choice between singular and regular metrics exists even inside the solution space of eq. (4.14). For scalar meson excitations with $L > 2$, vector mesons under suitable boundary conditions and all baryons, finally, confinement does not manifest itself in metric singularities at all.

Nevertheless, all higher meson and all baryon excitations are found to lie on the linear trajectories¹⁴ (3.7) and (3.8). The combination of these results may reflect the fact that the wave functions of light hadrons seem to be rather weakly affected by the linear confinement force. Indeed, several successful models for low-lying hadrons (e.g. models of Skyrme-type [37] and the instanton-based chiral quark model [49]) do not implement confinement at all since the mean separation among colored constituents appears to be too small for confinement effects to become relevant.

As noted in ref. [7], the abrupt hard wall singularity of the metric (2.8) — and the IR boundary conditions it requires — resemble those of an MIT bag model with a sharp surface [50]. In our case the singularities develop more gradually. The analogous bags therefore have smooth transition regions as they emerge dynamically in soliton bag models of Lee-Friedberg [51] or color-dielectric [52] types. Such soliton bags become confining by means of a space-dependent color dielectric function which induces singular couplings to

¹³It may be tempting to speculate about potential relations between these tachyon-induced singularities and closed string tachyon condensation [46] in the bulk, which is expected to be dual to confinement on the gauge theory side (see also refs. [47, 48]).

¹⁴The holographic models of refs. [11, 19] also realize linear meson trajectories with a non-singular metric, but they contain an additional (regular) dilaton field.

vacuum fields [53]. This suggests that the IR deformations of the dual gravity background found above encode information on the color-dielectric QCD vacuum structure.

String breaking due to light quark production is expected to stop the linear rise of the QCD confinement potential at large distances and hence to bend the linear hadron trajectories at sufficiently high excitation levels. Although such effects are not yet visible in the experimentally accessible part of the hadron spectrum, string breaking has recently been confirmed on the lattice [54]. In our model, however, the linearity of the spectra (3.7), (3.8) continues up to arbitrarily high excitation quanta (except for spin-0 mesons in $L = 0, 1$ states with $N \rightarrow \infty$, due to the finite z effects discussed above). This indicates that string breaking effects are absent in our holographic dual, as expected in the large- N_c limit (where N_c is the number of colors) or the associated weak coupling approximation on the gravity side.

A simple way to account for string breaking effects by hand would be to modify the solutions A at $z > z_m - \varepsilon$ such that the potential levels off. As already alluded to, for $(\lambda z_m)^2 \gg 1$ any reasonably smooth deformation of A at large z may in fact be implemented with practically no impact on the low-lying part of wavefunctions and spectra. In this way one could for example remove the singularities altogether. Not surprisingly, this also implies that the generation of the linear trajectories (1.1) does not fully constrain the IR behavior of the gravity background. Together with the second boundary condition in the meson sector, the remaining freedom could be used to implement a more comprehensive set of QCD observables.

5.2 L dependence

The IR deformations of the AdS₅ metric obtained from the solutions of eqs. (4.11), (4.14) and (4.21) are necessarily L dependent. This L dependence enters through the potentials (3.2), (3.3) which give rise to the inhomogeneities of the differential equations for $A_{S,V,B}$, and more universally through their counterparts (4.5)–(4.7) in the IR deformed background. Its ultimate source is therefore the L -dependent twist dimension of the considered hadron interpolators, imposed via eqs. (4.8)–(4.10), and its main effects are independent of the specific choice for the heuristic potentials or the replacement rule (3.1). A somewhat analogous hadron dependence of a dual background has been found in ref. [20], where vector and axial vector mesons feel a different metric, and would enter several other holographic models if observables in the whole hadron spectrum were to be reproduced.

A natural source for the L dependence arises in our approach from the identification of orbital hadron excitations as stringy quantum fluctuations about the AdS background [7, 16]. Indeed, such L dependent fluctuations may deform the AdS background metric in an L dependent fashion. In the simpler case of two-dimensional quantum gravity, fluctuation-induced deformations of a background metric (essentially AdS₂) were recently found explicitly [55]. In our case, the back-reaction of the metric to a fluctuation dual to a given orbital excitation could conceivably lead to analogous, L dependent deformations, as found in our solutions $A_{S,V,B}(z)$. Although the different orbital excitations *feel* a different total metric, however, the overall conformal symmetry breaking scale λ remains (almost)

hadron independent. This is a consequence of its relation (3.9) to the almost universal slope of all hadron trajectories, which we will discuss quantitatively in section 6.

The identification of orbital excitations as duals of metric fluctuations sets their holographic origin apart from that of the radial (i.e. N) excitations. This becomes manifest in the fact that our metric acquires no N dependence while even more sophisticated IR deformations would necessarily be L dependent since each orbitally excited hadron state is created by a different interpolator. Via back-reactions similar to those considered in refs. [48, 57] this L dependence may carry over to additional background fields of stringy origin as well. The relative weakness of the warp factors' L dependence in the physically dominant region $z < \sqrt{2}\lambda^{-1}$ and for larger L (where it becomes identical in the meson and baryon sectors) may be another indication for their fluctuation-induced origin.

Additional support for the above interpretation of the angular-momentum dependence in our background arises from an observation in ref. [11]. The latter asserts that for on-shell gauge theory properties which are described by the quadratic part of the dual string action, like the trajectories (1.1), the effect of higher derivative terms (including those related to orbital angular momentum operators, cf. section 2) can be essentially reproduced by the standard quadratic terms — to which we restrict ourselves here — in a modified gravity background. The modifications will depend, in particular, on the hadronic angular momentum carried by the dual modes, as manifested in our case in the solutions $A_{S,V,B}$.

Pursuing the above line of reasoning farther, we recall that in the holographic model based on the hard-wall metric (2.8) quantum fluctuations corresponding to orbital excitations are (at least in the conformal regime at small z) represented by an L dependent effective mass for the bulk string modes. Our derivation of the dual gravity backgrounds suggests a generalization of this interpretation. By allowing the effective masses to change with resolution $r = R^2/z$ outside of the conformal regime (i.e. for $z > 0$, as via the replacement (3.1)) one may be describing the quantum fluctuations of the metric and their potentially deforming back-reaction in more detail, and hence obtain a more accurate description of IR properties (including the linear hadron trajectories (1.1)) on the gauge theory side.

5.3 Background field content and underlying dynamics

The construction of our background geometry raises the question how it may be related to an underlying string theory. One could start to gain insight into this matter by examining, for example, whether the differential equations (4.11), (4.14) and (4.21) for the metric can be at least approximately cast into the form of (potentially higher-dimensional) Einstein equations. Such issues are beyond the scope of the present paper, however, where we focus on general principles, symmetries and experimental data to constrain the holographic dual in bottom-up fashion but leave the underlying dynamics at least *a priori* undetermined. In the remainder of this section we will therefore only mention a few qualitative aspects of the dynamics expected to govern the dual background and summarize the rationale for restricting our construction to the metric.

As stated in the introduction, the present knowledge of string theory in strongly curved spacetimes does not provide *ab initio* insight into the background field composition and

dynamics of the QCD dual. (Guidance from the supergravity approximation, in particular, is limited even at large N_c : it would predict a mass gap far smaller than the string tension beyond which linear trajectories can emerge, for example, in conflict with QCD phenomenology [11].) Nevertheless, several approximate, QCD-like duals, partly including fundamental flavor as mentioned in sections 1 and 2, have recently been obtained from brane constructions in which the metric and other background fields are solutions of classical field equations. Other dual models were derived by solving simple, string-inspired scalar and Einstein equations [48, 57] and used to get an idea of the impact of QCD condensates on the background. These models show, in particular, that such back-reactions can generate confining IR cutoffs in the fifth dimension similar to those encountered in our approach.

Our exclusive reliance on the dual background metric, finally, was guided by the dimensional power-counting argument of section 4, the possibility to represent the impact of additional background fields on the resonance masses by deformations of the metric (cf. section 5.2), and by an Occam-razor type preference for the minimal and hence most efficient background required to reach our objectives. Nevertheless, experience from brane models as discussed above suggests that a more comprehensive and detailed approximation to the QCD dual should contain additional background fields. As pointed out in ref. [11], for example, one may expect a background including tachyon and dilaton fields if the dual confinement mechanism has its origin in closed-string tachyon condensation (see also ref. [48]).

5.4 Comparison with other confining holographic models

In the following we will briefly compare our holographic model to a few related approaches which also contain dual representations of linear confinement and linear trajectories in the hadron spectrum. As already mentioned, it turns out to be a nontrivial task to reproduce linear trajectories with approximately universal slopes not only in the meson but also in the baryon channels. In fact, our approach seems to be the first which accomplishes this. Hence our comparisons below have to remain restricted to the meson sector.

Linear “meson” trajectories in more or less QCD-like gauge theories were e.g. found in refs. [23, 42]. A recent implementation of linear trajectories for both radial and spin excitations of the rho meson into the AdS/QCD framework [11] induces conformal symmetry breaking mainly by a dilaton background field. In the simplest case a dilaton of the form $\Phi(z) \propto z^2$ is added to the pure AdS metric and hence is exclusively responsible for the non-conformal IR behavior. (Essentially the same term was argued to arise from a dual magnetic condensate (which plays the role of a Higgs field) in the partition function of a QCD instanton ensemble when promoted into the bulk [56].) Although this approach works well in the (vector) meson sector, we have already noted that such dilaton effects do not manifest themselves in the baryon spectrum since they can be absorbed into the eigenmodes and leave the AdS/CFT boundary condition unchanged. An interesting observation of ref. [11] is that the exponent of the warp factor should not contain contributions growing as z^2 for $z \rightarrow \infty$, in order to have spin-independent slopes of the radial rho meson

excitation trajectories. Our solutions $A_{S,V,B}$ grow logarithmically with z for large¹⁵ z and are therefore consistent with this condition (while the model of refs. [19, 42] is not, see below).

In other recent work, scalar bulk fields dual to the gluon and bilinear quark condensate operators have been shown to generate — by their back-reaction on the gravity background — a confining restriction of the metric to a deformed AdS slice [48, 57]. Indications for the potential role of the two-dimensional QCD condensate [45] in the confinement mechanism arise in these frameworks as well. As already mentioned, the non-conformal behavior of our metric towards small z , induced by the confinement-generating modifications of the string mode potentials, may similarly be a reflection of the non-local dimension-two condensate. This suggests a more general analysis of the information on QCD condensates and more specific IR degrees of freedom (e.g. topological ones related to instantons, monopoles or center vortices) [58] which is encoded in our background metric.

The qualitative small- z behavior $\exp(cz^2)$ of our solutions for the non-conformal warp factors is identical to that proposed in refs. [19, 42] where it has been shown to embody, together with a constant dilaton, a linearly growing heavy-quark potential and a linear (mesonic) Regge trajectory [19, 42]. The value of c was estimated to be $c \sim -0.9 \text{ GeV}^2$ (in a metric with Lorentzian signature) which is in the same ballpark as ours for intermediate L . Indeed, anticipating the relation (6.5) between λ and Λ_{QCD} to be established in section 6, our warp factor implies e.g. $c_{S,L=2} \simeq -0.7 \text{ GeV}^2$. Our values for c_B in the baryon sector are positive, however, which may suggest some differences in the dual confinement mechanisms for vector mesons and baryons.

6. Phenomenological implications

In the following section we proceed to the quantitative analysis of our holographic dual and confront the predicted mass spectra (3.7), (3.8) for the light hadrons with experimental data. Recent reviews of excited hadrons, their symmetry structure, parity doubling etc. can be found in refs. [59].

We start by determining the conformal symmetry breaking scale λ from data for the slope $W = 4\lambda^2$ of the trajectories (1.1). Fits to the experimental meson spectra yield $W = (1.25 \pm 0.15) \text{ GeV}^2$ [27] and $W = (1.14 \pm 0.013) \text{ GeV}^2$ [28]. These values allow for an immediate check of our relation (3.10) which predicts the rho meson mass as a function of its trajectory slope, i.e.

$$M_\rho = \sqrt{\frac{W}{2}}. \tag{6.1}$$

The above empirical results for W imply $M_\rho = 0.79 \text{ GeV}$ or $M_\rho = 0.76 \text{ GeV}$, respectively, which are both consistent with the experimental value $M_\rho = 0.7755 \pm 0.0004 \text{ GeV}$ [60]. Since the latter is close to the mean of the slope fit results, we choose the experimental rho mass to set the scale of the deformed gravity background, i.e.

$$\lambda = \sqrt{\frac{W}{4}} = \frac{M_\rho}{\sqrt{2}} = 0.55 \text{ GeV}. \tag{6.2}$$

¹⁵Except if the large- z region is cut off by a singularity, of course

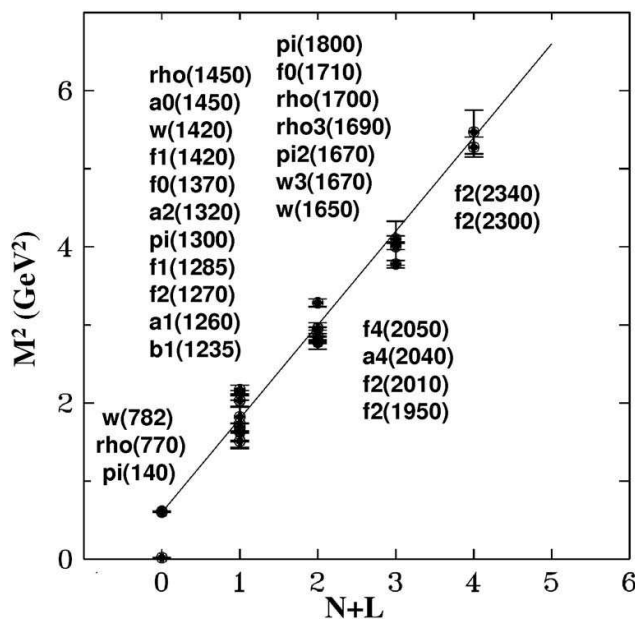


Figure 3: Experimental meson mass spectrum from ref. [60] and the predicted trajectory for $W = 2M_\rho^2 \simeq 1.01 \text{ GeV}^2$.

The corresponding value $W = 4\lambda^2 = 1.21 \text{ GeV}^2$ fixes the slope of our linear meson trajectory which is compared to the experimental meson resonance spectrum (for quark-antiquark states) in figure 3.

The clustering of radial and orbital excitations is clearly visible, and even the highest radial excitations $f_2(2300)$ and $f_2(2340)$ lie squarely on the linear trajectory. The pion ground state, set apart by its approximate Goldstone boson nature, does not fit into the overall pattern predicted by the dual string modes. This problem is expected. It was already encountered in ref. [7] and is caused by the lack of chiral symmetry and its breaking in our approximate holographic dual. For the same reasons, qualitative models for the light-front square mass operator in the mesonic valence quark sector [61] (which reproduce the radial excitation trajectory) need a strong additional short-range attraction in the spin-0 channel to reproduce the π - ρ mass splitting (for $L = 0$). Since the dual string modes are related to the valence components of the light-front wave function (with z playing the role of a relative coordinate) [21], the impact of such interactions is accessible in our approach.

The empirical slope of the Δ trajectory (including the nucleon resonances in the 4_8 representation of $SU(4)$) is $W = (1.081 \pm 0.035) \text{ GeV}^2$ [26]. As in the meson sector, our relation (3.11) turns this value into a prediction for the Δ ground state mass,

$$M_\Delta = \sqrt{\frac{3W}{2}} = 1.27 \text{ GeV}, \quad (6.3)$$

which compares well with the experimental value $M_\Delta = 1.232 \text{ GeV}$. This suggests to use

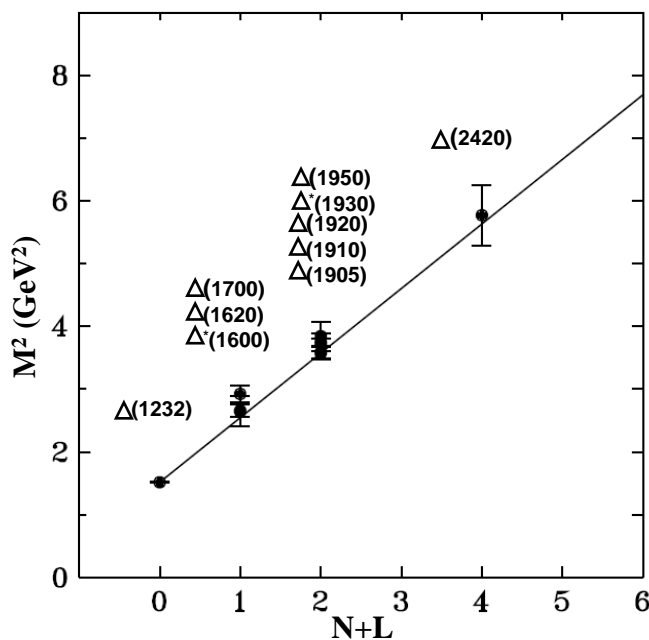


Figure 4: Experimental Delta isobar mass spectrum from ref. [60] and the predicted trajectory for $S = 3/2$ (in the 4_8 representation of $SU(4)$) with $W = 2M_{\Delta}^2/3 \simeq 1.01 \text{ GeV}^2$.

the experimental mass of the Δ isobar for an alternative determination of the scale

$$\lambda = \frac{M_{\Delta}}{\sqrt{6}} = 0.50 \text{ GeV} \tag{6.4}$$

in the baryon sector. The value (6.4) differs by less than 10% from that based on the experimental rho mass, eq. (6.2). This confirms the approximate universality of λ and of the associated slopes $W = 4\lambda^2$ in the meson and baryon sectors.

The resulting Δ isobar trajectory, together with the empirical square masses of the radial and orbital resonances, is shown in figure 4. The first radial excitations, $\Delta(1600)_{\frac{3}{2}^+}$ (with $L = 0$) and $\Delta(1930)_{\frac{5}{2}^-}$ (with $L = 1$) are degenerate with states carrying one or two units of angular momentum, respectively. The parity doublets $\Delta(1600)_{\frac{3}{2}^+}$, $\Delta(1700)_{\frac{3}{2}^-}$ and $\Delta(1905)_{\frac{5}{2}^+}$, $\Delta(1930)_{\frac{5}{2}^-}$, furthermore, are states which differ by one radial and one orbital excitation quantum such that $N + L$ is preserved.

Finally, we turn to the nucleon and its excitations. As noted in ref. [26], the trajectory of the nucleon resonances in the 2_8 representation of $SU(4)$ (including the nucleon itself) lies below that for the 4_8 representation. This behavior can be accommodated by our covariant¹⁶ framework with a somewhat smaller value of $\lambda = 0.47 \text{ GeV}$ which may e.g. be due to hyperfine interactions. The resulting trajectory indeed fits the nucleon resonances in the 2_8 representation well and is shown as a solid line in figure 5. The experimental

¹⁶Note that the identification of the not separately Lorentz-invariant orbital angular momentum L requires us to select a particular frame, which underlies the interpretation of our interpolators (cf. section 2).

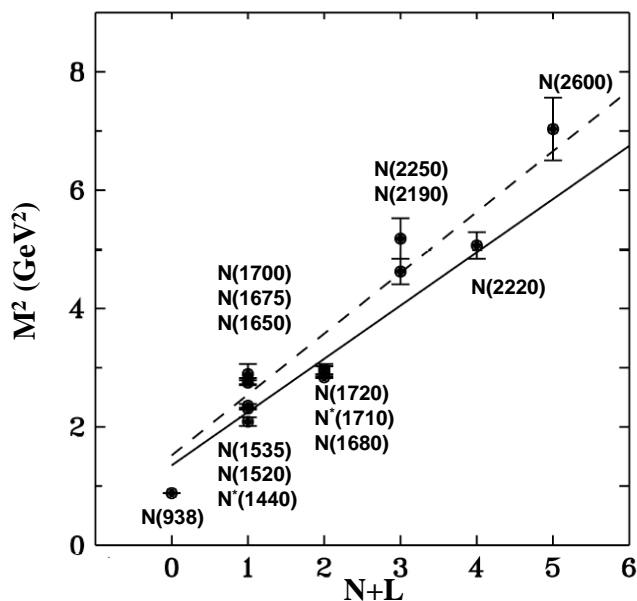


Figure 5: Experimental nucleon mass spectrum from ref. [60] and the predicted trajectories for $S = 1/2$ with $W \simeq 0.9 \text{ GeV}^2$ (solid line) and for $S = 3/2$ with $W = 2M_\Delta^2/3 \simeq 1.01 \text{ GeV}^2$ (dashed line).

value of the nucleon mass lies below the value $M_N = 1.16 \text{ GeV}$ on the trajectory, however. This may again be related to chiral symmetry breaking effects and implies, in any case, that the resulting nucleon delta splitting vanishes. Since the latter is an $O(1/N_c)$ effect (where N_c is the number of colors), this result is consistent with the large- N_c limit, i.e. the weak string coupling limit which underlies all known top-down holographic duals. The trajectory of figure 4 is included as a dashed line in figure 5 and seen to fit the nucleon resonances in the 4_8 representation, as anticipated.

Our holographic results for the at present experimentally accessible orbitally excited nucleon states are generally close to those of ref. [7] (which were based on the hard IR wall metric (2.8)) because at moderate $N + L$ the difference between linear and quadratic trajectories is rather small. In addition, we predict the radially excited states, i.e. the Roper resonance $N(1440)$ and the second radially excitation $N(1710)$, which is almost degenerate with the $L = 2$ states $N(1680)$ and $N(1720)$. Our results for nucleons with internal spin $3/2$ lie approximately on the Δ trajectory (dashed line), as already mentioned. The parity doubling of baryon states with fixed total spin, differing by one unit of angular momentum and one internal spin or radial excitation quantum, emerges naturally in our approach.

The slope W of our trajectories is related to the QCD scale. As a consequence of confinement, the Gaussian suppression factor $\exp[-(W/8)z^2]$ prevents the dual string modes (3.4) - (3.6) from extending significantly beyond distances $z_m \sim \sqrt{8/W}$ into the fifth dimension. Similar confinement effects are often modelled by the hard IR wall metric (2.8)

with $z_m \sim \Lambda_{\text{QCD}}^{-1}$, as discussed in section 4. Hence we approximately identify

$$\Lambda_{\text{QCD}} \simeq \sqrt{\frac{W}{8}} = \frac{\lambda}{\sqrt{2}} \simeq 0.35 \text{ GeV}. \tag{6.5}$$

The numerical estimate in eq. (6.5) is based on the phenomenological slopes of about 1 GeV^2 and indeed close to the empirical value $\Lambda_{\text{QCD}} \simeq 0.33 \text{ GeV}$ (at hadronic scales with three active flavors) [60]. Finally, we recall that the string tension resulting from the semiclassical treatment of simple, relativistically rotating string models [22], i.e.

$$\sigma = \frac{W}{2\pi} \simeq 0.88 \frac{\text{GeV}}{\text{fm}}, \tag{6.6}$$

is consistent with standard values as well.

7. Summary and conclusions

We have shown how a salient empirical pattern in the light hadron spectrum, namely the combination of both radial and orbital excitations into linear trajectories of approximately universal slope, can be reproduced with good accuracy by a rather minimal version of holographic QCD. Our approximate holographic dual relies exclusively on IR deformations of the AdS metric, governed by one free mass scale proportional to Λ_{QCD} , and generates the mass gap expected from confining gauge theories. Moreover, it provides the first example of a gravity dual which is able to reproduce linear trajectories in the baryon sector as well.

The resulting light hadron spectra are in good overall agreement with the available experimental data for both meson and baryon masses. Discrepancies between the radial and orbital resonance masses in the hard-wall model are resolved and the experimentally established, approximately universal slope of the trajectories emerges naturally. Moreover, new relations between the ρ meson and Δ isobar ground state masses and the slopes of their respective trajectories are predicted. Since the linearity of all trajectories extends to the lightest masses, however, they fail to reproduce the physical pion and nucleon ground states. This is not unexpected because our approximate gravity dual in its present form lacks information on chiral symmetry and residual interactions responsible e.g. for hyperfine splittings.

Our holographic background was derived by reconstructing the dual mode dynamics from spectral properties on the gauge theory side. The underlying strategy may be useful for other applications as well. It consists of first finding the modifications of the AdS string mode potentials which generate a desired gauge theory result, and to subsequently construct the corresponding fields on the gravity side by equating the potentials induced by a general background to their heuristic counterparts. As long as a dual background exists, its derivation is then reduced to solving the resulting differential equations.

Above we have constructed the holographic duals for radial and orbital hadron trajectories in a minimal way, i.e. by a non-conformal warp factor. A remarkable *a posteriori* justification for this restriction is its sufficiency. More complex IR deformations and further bulk fields would also introduce new parameters to be fixed by QCD phenomenology

and hence lessen the predictivity of the dual description. In the baryon sector we were able to derive the resulting IR deformations of the metric analytically. The lowest orbital excitations of the spin-0 mesons, and depending on a boundary condition also the higher orbital vector meson excitations, encounter a singular metric. This is a dual confinement signature and results in a dynamical compactification of the fifth dimension, hence directly linking linear trajectories (for not too high excitation levels) to linear quark confinement.

Despite the advantages of the minimal description, however, experience from supergravity and brane models suggests that a more comprehensive holographic dual may require a more general form of the metric and additional background fields. The prospect of deriving those by our method deserves further investigation. Among the potentially useful extensions and applications we mention the implementation of quark flavor and spontaneously broken chiral symmetry as well as the calculation of condensates, heavy-quark potentials and light-front wave functions.

Acknowledgments

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A. Solutions of the linearized differential equations for the mesonic warp factor

In this appendix we solve the linearized versions of the differential equations (4.14) and (4.21) for the non-conformal mesonic warp factors. The results will be useful for our analysis of the solution behavior of the full, nonlinear equations in section 4. It will also be instructive to understand the differences in the solution behavior induced by tachyonic, massless and massive modes in this simplified setting, although the solutions of the linearized equations do not develop finite- z singularities.

The linearization of the differential equation (4.14) for the spin-0 meson sector leads to the (still inhomogeneous) equation

$$z^2 A'' - 3z A' + \frac{4}{3} (L^2 - 4) A - \frac{2}{3} \lambda^2 z^2 (\lambda^2 z^2 + 2L) = 0 \tag{A.1}$$

whose solutions provide approximations to those of the full equation in the regions where $A \ll 1$. As a consequence of the conformal boundary condition $A(0) = 0$, which turns out to be an automatic property of all solutions which stay finite at $z = 0$, this condition should hold in particular in the UV, i.e. for z close to zero.

The full solution space of the linear equation (A.1) can be constructed by Frobenius expansion techniques (for the homogeneous part) and by guessing special solutions of the

full, inhomogeneous equation or by deriving them with the help of Green function methods. Either way, the general solution for $L = 0, 1$ (associated with the tachyonic string modes in the bulk) is found to be

$$\bar{A}_{S,L=0,1}(z) = \frac{L\lambda^2 z^2}{L^2 - 7} + \frac{\lambda^4 z^4}{2L^2 - 8} + c_1 (\lambda z)^{2+2\sqrt{\frac{7-L^2}{3}}} + c_2 (\lambda z)^{2-2\sqrt{\frac{7-L^2}{3}}}. \quad (\text{A.2})$$

This solution satisfies the initial condition $\bar{A}(0) = 0$ only for $c_2 = 0$. The remaining irrational-power term is subleading at small z both for $L = 0$ and $L = 1$. For $L = 2$, the general solution (induced by the massless string mode) is

$$\bar{A}_{S,L=2}(z) = -\frac{2}{3}\lambda^2 z^2 - \frac{1}{24}(1 + c_1)\lambda^4 z^4 + \frac{1}{6}\lambda^4 z^4 \ln \lambda z + c_2 \quad (\text{A.3})$$

where the initial condition $\bar{A}(0) = 0$ requires $c_2 = 0$. For $L > 2$, finally, the particular solution of the inhomogeneous equation is identical to that for $L = 0, 1$ while the general solution of the homogeneous equation differs. Their sum,

$$\begin{aligned} \bar{A}_{S,L>2}(z) = & \frac{L\lambda^2 z^2}{L^2 - 7} + \frac{\lambda^4 z^4}{2L^2 - 8} \\ & + c_1 \lambda^2 z^2 \cos\left(2\sqrt{\frac{L^2 - 7}{3}} \ln \lambda z\right) + c_2 \lambda^2 z^2 \sin\left(2\sqrt{\frac{L^2 - 7}{3}} \ln \lambda z\right), \end{aligned} \quad (\text{A.4})$$

is the general solution of equation (A.1) and satisfies the initial condition $\bar{A}(0) = 0$ for all (finite) values of $c_{1,2}$. Note that for $L = 0, 1$ the coefficients of both $\lambda^2 z^2$ and $\lambda^4 z^4$ are negative, for $L = 2$ the coefficient of $\lambda^2 z^2$ is negative and that of $\lambda^4 z^4$ can be chosen positive (so that $\bar{A}_{L=0} > 0$ for larger z), and for $L > 2$ both coefficients are positive.

The analysis of the linearized version

$$-z^2 A'' - 3z A' + \frac{4}{3}(L^2 - 1)A - \frac{2}{3}\lambda^2 z^2 (\lambda^2 z^2 + 2L) = 0 \quad (\text{A.5})$$

of equation (4.21) in the vector meson channel proceeds analogously. For $L = 0$ it yields the general solution

$$\bar{A}_{V,L=0}(z) = -\frac{2}{7}\lambda^2 z^2 - \frac{1}{38}\lambda^4 z^4 + c_1 (\lambda z)^{-1} \cos\left(\frac{\ln \lambda z}{\sqrt{3}}\right) + c_2 (\lambda z)^{-1} \sin\left(\frac{\ln \lambda z}{\sqrt{3}}\right) \quad (\text{A.6})$$

where $\bar{A}_{V,L=0}(0) = 0$ demands $c_1 = c_2 = 0$. For $L = 1$ one finds

$$\bar{A}_{V,L=1}(z) = -\frac{1}{3}\lambda^2 z^2 - \frac{1}{36}\lambda^4 z^4 + c_1 \frac{1}{\lambda^2 z^2} + c_2 \quad (\text{A.7})$$

where $\bar{A}_{V,L=1}(0) = 0$ again requires $c_1 = c_2 = 0$. The solution for $L > 1$, finally, is

$$\bar{A}_{V,L>1}(z) = \frac{L\lambda^2 z^2}{L^2 - 7} + \frac{\lambda^4 z^4}{2L^2 - 38} + c_1 (\lambda z)^{-1-\sqrt{\frac{4L^2-1}{3}}} + c_2 (\lambda z)^{-1+\sqrt{\frac{4L^2-1}{3}}} \quad (\text{A.8})$$

where $\bar{A}_{V,L>1}(0) = 0$ demands $c_1 = 0$ while c_2 remains unconstrained. For $L = 2$ the irrational power term provides the leading small- z behavior. The coefficient of the $\lambda^2 z^2$ ($\lambda^4 z^4$) term turns positive for $L \geq 3$ ($L \geq 4$).

The inhomogeneities in eqs. (A.1) and (A.5) determine the small- z behavior of the solutions (with the exception of $\bar{A}_{V,L=2}$). In our context, this has two pertinent consequences. First, the leading small- z dependence is generally determined by the special solutions of the inhomogeneous equation and therefore completely fixed, i.e. only the subleading small- z behavior depends on the boundary conditions.

A second useful consequence of the inhomogeneities in eqs. (A.1) and (A.5) — which are the same as those in eqs. (4.14) and (4.21) — is related to the fact that the leading small- z behavior of the solutions to the full equations is identical to that of their linearized counterparts. Hence at small z the modulus of all solutions with $A(0) = 0$ grows as $\lambda^2 z^2$ (except for $\bar{A}_{S,L=0}$ and $\bar{A}_{V,L=2}$), partially with trigonometric or logarithmic corrections.

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